

# Quantum cosmology of 5D non-compactified Kaluza-Klein theory

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## Abstract

We study the quantum cosmology of a five dimensional non-compactified Kaluza-Klein theory where the 4D metric depends on the fifth coordinate,  $x^4 \equiv l$ . This model is effectively equivalent to a 4D non-minimally coupled dilaton field in addition to matter generated on hypersurfaces  $l = \text{constant}$  by the extra coordinate dependence in the four-dimensional metric. We show that the Vilenkin wave function of the universe is more convenient for this model as it predicts a new-born 4D universe on the  $l \simeq 0$  constant hypersurface.

## 1 Introduction

The subject of initial conditions is one of the most important questions in cosmological models. Unlike a classical system where the dynamical equations are solved subject to the initial conditions, for cosmological models there are no initial conditions external to the universe to be considered for solving the Einstein equations. This is because there is no time parameter external to the universe. We know that the issue of initial conditions in classical cosmology corresponds to a boundary condition problem in quantum cosmology. Therefore, it seems that

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the initial conditions should be introduced, from the outside, within some boundary conditions. Two well-known proposals commonly used in the literature are the Hartle-Hawking *no boundary* proposal [1]-[4], and the Vilenkin *tunneling* proposal [5]-[9]. The first proposal is that the universe has no boundary in 4D Euclidean space and the second one states that only the outgoing modes of the wave function should be taken at the singular boundary of superspace. Some attempts to generalize these proposals to higher-dimensional Kaluza-Klein cosmologies, to find a reasonable explanation to the large separation of the scale of the observed three-dimensional universe and the scale of the extra dimensions have already been done [10], [11]. In these works, the extra dimensions were supposed to be stable and compactified by a cyclic symmetry to a small size.

In this paper, we follow another approach and investigate the quantum cosmology of a non-compactified Kaluza-Klein theory developed by Wesson and co-workers [12]-[15]. Unlike the usual Kaluza-Klein theory in which a cyclic symmetry associated with the extra dimension is assumed, the new approach removes the cyclic condition on the extra-dimension, and derivatives of the metric with respect to the extra-coordinate are retained. This induces non-trivial matter on the hypersurfaces of  $l = \text{const.}$

Our goal is to investigate the effect of the  $l$ -dependence of the metric on the quantum cosmology of a simple model and obtain the relevant initial condition on the fifth coordinate. We find that while the Vilenkin wave function leads to probability distribution of quantum tunneling peaked around the  $l \simeq 0$  hypersurface, the Hartle-Hawking wave function leads to large  $l$  values corresponding to highest probability for the birth of a Lorentzian universe. This leads us to choose the Vilenkin wave function as the more convenient one for the non-

compactified Kaluza-Klein theory, since it seems unnatural that the 4-dimensional universe was born very far from the  $l \simeq 0$  constant hypersurface.

## 2 Wheeler-DeWitt equation

In this non-compactified Kaluza-Klein theory we choose the general 5-dimensional metric as:

$$ds^2 = -\hat{N}^2(t, l)e^{-b\sigma(t)}dt^2 + \frac{\hat{a}^2(t, l)e^{-b\sigma(t)}}{[1 + \frac{1}{4}kr^2]^2}dx^i dx^i + \epsilon e^{2b\sigma(t)}dl^2. \quad (1)$$

Here  $l$  is the fifth coordinate, and  $\hat{N}(t, l) = N(t)f(l)$ ,  $\hat{a}(t, l) = a(t)\chi(l)$  are the  $l$ -dependent separable lapse function and scale factor respectively. Also  $k = 0, \pm 1$  is the curvature,  $b$  is a parameter,  $\sigma(t)$  is a dilaton field and  $\epsilon = \pm 1$  which leaves the signature of the fifth dimension general. The  $l$ -dependence of the 4D geometry, namely  $\hat{N}(t, l)$  and  $\hat{a}(t, l)$  indicates that the cyclic condition on the fifth coordinate is removed. The Ricci curvature scalar is calculated to be

$$\hat{\mathcal{R}} = \left[ -6\frac{\ddot{\hat{a}}}{\hat{N}^2\hat{a}} + b\frac{\ddot{\sigma}}{\hat{N}^2} - 6\frac{\dot{\hat{a}}^2}{\hat{N}^2\hat{a}^2} - \frac{3b^2}{2}\frac{\dot{\sigma}^2}{\hat{N}^2} + 3b\frac{\dot{\hat{a}}\dot{\sigma}}{\hat{N}^2\hat{a}} - 6\frac{k}{\hat{a}^2} \right] e^{b\sigma} + 6\epsilon \left[ \frac{\hat{N}'\hat{a}'}{\hat{N}\hat{a}} + \frac{\hat{N}''}{3\hat{N}} + \frac{\hat{a}'^2}{\hat{a}^2} + \frac{\hat{a}''}{\hat{a}} \right] e^{-2b\sigma}. \quad (2)$$

Inserting the scalar curvature into the 5D vacuum Einstein-Hilbert action

$$S = \int dx^5 \sqrt{-\hat{g}} \hat{\mathcal{R}},$$

leads to the following effective action

$$S = \int \hat{L} dt,$$

with

$$\hat{L} = \hat{N} \left[ \frac{\hat{a}\dot{\hat{a}}^2}{2\hat{N}^2} - \frac{b^2}{8}\frac{\hat{a}^3\dot{\sigma}^2}{\hat{N}^2} - \frac{1}{2}k\hat{a} + \frac{\epsilon}{2}e^{-3b\sigma}\hat{a}^3 F(l) \right]. \quad (3)$$

Here

$$F(l) = \frac{f'\chi'}{f\chi} + \frac{f''}{3f} + \frac{\chi'^2}{\chi^2} + \frac{\chi''}{\chi}, \quad (4)$$

and the dilaton field potential is

$$U(\sigma) = \epsilon e^{-3b\sigma} F(l). \quad (5)$$

The Hamiltonian form of the action can be written

$$S = \int (\hat{P}_a \dot{\hat{a}} + P_\sigma \dot{\sigma} - \hat{N} \hat{H}) dt,$$

where  $\hat{N}$  appears as the Lagrange multiplier. The variation of the action with respect to  $\hat{N}$  leads to the Hamiltonian constraint

$$\hat{H} = \frac{\hat{P}_a^2}{2\hat{a}} - \frac{2}{b^2} \frac{P_\sigma^2}{\hat{a}^3} + \frac{1}{2} k \hat{a} - \frac{1}{2} \hat{a}^3 U(\sigma) = 0. \quad (6)$$

Since no lapse function  $N(t)$  appears in the Hamiltonian, it is an indication of the invariance of the Hamiltonian under time reparametrization. This means  $N(t)$  has no physical importance and we may choose the common gauge in cosmology, namely  $N(t) = 1$ . The Wheeler-DeWitt equation in the minisuperspace of coordinates  $0 < R < \infty, -\infty < \sigma < \infty$  can then be written as

$$\left[ -\frac{1}{\hat{a}^p} \frac{\partial}{\partial \hat{a}} \hat{a}^p \frac{\partial}{\partial \hat{a}} + \frac{4}{b^2 \hat{a}^3} \frac{\partial^2}{\partial \sigma^2} + k \hat{a} - \hat{a}^3 U(\sigma) \right] \Psi(\hat{a}, \sigma) = 0. \quad (7)$$

Here  $p$  covers the ambiguity in factor-ordering, but since we will restrict ourselves to the semiclassical approximation omitting the first derivatives, this factor is not important [17]. Then, the Wheeler-DeWitt equation can be rewritten as

$$\left[ -\frac{\partial^2}{\partial \hat{a}^2} + \frac{4}{b^2 \hat{a}^2} \frac{\partial^2}{\partial \sigma^2} + W(\hat{a}, \sigma) \right] \Psi(\hat{a}, \sigma) = 0, \quad (8)$$

where

$$W(\hat{a}, \sigma) = \hat{a}^2[k - \hat{a}^2U(\sigma)] \quad (9)$$

is the superpotential. In the investigation of the Wheeler-DeWitt equation we need to know the properties of the superpotential  $W(\hat{a}, \sigma)$ . For fixed  $\sigma$  the superpotential consists of two terms, a curvature term  $k\hat{a}$  and the term  $\hat{a}^3U(\sigma)$ , where  $U(\sigma)$  acts effectively as a cosmological term. By appropriate choices for  $k$  and  $\epsilon$  the superpotential may have a maximum necessary for quantum tunneling. However, along the line of fixed  $\hat{a}$ , the potential  $U(\sigma)$  has no a maximum. This would suggest that we may concentrate on the  $\hat{a}$  coordinate as a viable dynamical variable in the investigation of quantum tunneling in the  $\hat{a}$  direction, and consider the coordinate  $\sigma$  as a parameter. We will discuss this subject in the next section.

The relevant classical cosmology subject to quantization is a closed universe with  $k = +1$ . This is because the universes with  $k = -1, 0$  lead to an infinite volume in the integration of the action and so the nucleation probability of the universe in the quantum creation process would be zero. Thus, we take a closed universe,  $k = +1$ . The superpotential (9) exhibits a barrier if the dilaton potential (5) is greater than zero

$$U(\sigma) > 0,$$

or (if  $\epsilon = 1$ )

$$F(l) > 0.$$

In the semiclassical approximation we want to find the most probable initial conditions for the classical motion of the universe. This motion is controlled by the superpotential (9). To this

end, we write

$$\hat{a} = \hat{a}_0(\sigma) = \frac{1}{\sqrt{U(\sigma)}}, \quad (10)$$

and so define a surface of constant superpotential  $W = 0$  in the minisuperspace. In fact, equation (9) describes a superpotential barrier in the  $\hat{a}$  direction and equation (10) separates the under-barrier region  $0 < \hat{a} < \hat{a}_0$  from the outer region  $\hat{a} > \hat{a}_0$ . From (10) we can also obtain an equation for  $f(l)$  and  $\chi(l)$ :

$$\chi\chi' \frac{f'}{f} + \frac{\chi^2}{3} \frac{f''}{f} + \chi'^2 + \chi\chi'' = \text{const.} \quad (11)$$

The presence of a barrier region indicates that we can consider the nucleation of the universe through a quantum tunneling effect as discussed above. Therefore, we proceed to consider the well-known tunneling condition of Vilenkin [9]. Our aim is to find the approximate analytic solutions of the Wheeler-DeWitt equation, under the barrier and beyond the barrier.

### 3 Wave function

First, we show a behaviour of the “*nothing state*” for

$$\hat{a}^2 \ll \hat{a}_0^2.$$

According to Halliwell [17], in order to have a regular solution for the wave function in the limit  $\hat{a} \rightarrow 0$ , the wave function should be  $\sigma$ -independent because the coefficient of  $\partial_\sigma^2$  in the Wheeler-DeWitt equation (8) diverges. The Wheeler-DeWitt equation then reduces to

$$\left[ -\frac{d}{d\hat{a}^2} + \hat{a}^2 \right] \Psi(\hat{a}) = 0. \quad (12)$$

Introducing the auxiliary variable  $\Gamma(\hat{a}) = \Psi(\hat{a})/\hat{a}^{1/2}$  and the transformation  $\nu = \hat{a}^2/2$ , equation (12) reduces to the modified Bessel equation

$$\nu^2 d_\nu^2 \Gamma + \nu d_\nu \Gamma - \left( \nu^2 + \frac{1}{16} \right) \Gamma = 0, \quad (13)$$

whose independent solutions are the well-known modified Bessel functions of order  $1/4$ ,  $I_{1/4}(\nu)$ , and  $K_{1/4}(\nu)$ . Transforming to the old variables, we find the growing solution  $\hat{a}^{1/2} I_{1/4}(\hat{a}^2/2)$  and the decreasing solution  $\hat{a}^{1/2} K_{1/4}(\hat{a}^2/2)$  in the  $\hat{a}$  direction. To select one of them we will impose a matching condition with the solution (18) for  $\hat{a} \ll \hat{a}_0$ . This gives the decreasing solution

$$\Psi(\hat{a}) = \hat{a}^{1/2} K_{1/4} \left( \frac{\hat{a}^2}{2} \right), \quad (14)$$

which is the well-known solution of *nothing* due to Vilenkin [8], and goes like  $e^{-\frac{\hat{a}^2}{2}}$  for  $\hat{a} \rightarrow 0$ . It was obtained by Vilenkin in the limit of small  $\hat{a}$  in the 4-dimensional model with topology  $R \times S^3$  and inflation, without a dilaton field. Usually, in 4-dimensional cosmology *nothing* is the nonsingular boundary of the superspace that includes three-geometries given through a slicing of a regular four-geometry [8], [9]. In higher-dimensional cosmologies, the extra dimensions usually play the role of a scalar field  $\sigma$  in the equivalent four-dimensional model, such that the non-singular boundary of the minisuperspace is the configuration  $\hat{a} = 0, |\sigma| < \infty$ . This configuration is called *external nothing* since the extra dimension is assumed to be nonzero [11].

Now, consider the common Wheeler-DeWitt equation in 4D

$$\left[ -\frac{\partial^2}{\partial a^2} + \frac{1}{a^2} \frac{\partial^2}{\partial \sigma^2} + a^2 - a^4 U(\sigma) \right] \Psi(a, \sigma) = 0. \quad (15)$$

The WKB solution of Eq.(15) with Vilenkin boundary condition is well known [9] in the region

of the minisuperspace where the potential  $U(\sigma)$  is *sufficiently flat*, i.e. where

$$\left| \frac{U'}{U} \right| \ll \max\{U(\sigma), a^{-2}\}. \quad (16)$$

In fact, by assuming the condition (16) the wave function becomes a slowly-varying function of  $\sigma$ . Therefore, one can neglect the derivative with respect to the dilaton field  $\sigma$ . Thus,  $\sigma$  plays the role of a parameter in the Wheeler-DeWitt equation (15), and the problem is reduced to the one-dimensional minisuperspace model.

In the present model, however, the potential  $U(\sigma) = \epsilon F(l)e^{-3b\sigma}$  has a strongly asymmetric form for  $\sigma < 0$  and  $\sigma > 0$ , and so the condition (16) does not hold in the region of the minisuperspace where  $\hat{a}^2 U(\sigma) > 1$  and  $\sigma \ll 0$  with  $b > 0$ . Nevertheless, we may use an approximation to cast the Wheeler-DeWitt equation (8) into a solvable equation. The main point in the non-compactified five-dimensional model is to find the most probable  $l = \text{const}$  hypersurface for the 4D universe to tunnel from *nothing*. On the other hand, we are interested in the tunneling in the direction of  $\hat{a}$  which is the only  $l$ -dependent dynamical variable in the model. Therefore, any result about the most probable  $l = \text{const}$  hypersurface will be obtained due to the  $l$ -dependence of  $\hat{a}$ , and is expected not to be affected by the  $\sigma$  dependence of  $\Psi$  since  $\sigma$  is independent of  $l$ . So, we may solve the Wheeler-DeWitt equation in that region of the minisuperspace where the condition (16) holds and deduce the result about the most probable  $l = \text{const}$  hypersurface for tunneling. This result is anticipated not to be changed if we solve the Wheeler-DeWitt equation in the whole minisuperspace. In other words, to the extent that we are concerned about finding the most probable  $l = \text{const}$  hypersurface ( and the dependence of  $\Psi$  on the  $\sigma$  field is not important to us ) we may suppose the wave function  $\Psi$  to be independent of  $\sigma$ . This assumption is valid at least in the positive sector  $\sigma \gg 0$  with

$b > 0$  where the potential  $U(\sigma)$  is sufficiently flat. Therefore, we may drop the  $\sigma$  derivative and the Wheeler-DeWitt equation (8) takes the form

$$\left[ -\frac{d}{d\hat{a}^2} + \hat{a}^2[1 - U(\sigma)\hat{a}^2] \right] \Psi(\hat{a}) = 0, \quad (17)$$

where  $\sigma$  is just a parameter and the problem is essentially identical to the one-dimensional minisuperspace model with the dynamical variable  $\hat{a}$ . In this approximation, the superpotential barrier becomes wide for  $\sigma \gg 0$  which, at first glance, makes it inconvenient for tunneling. However, due to the function  $F(l)$  we will show that the barrier becomes sufficiently narrow for quantum tunneling, even in the  $\sigma \gg 0$  sector of the minisuperspace. The solutions of equation (17) are the well-known Vilenkin tunneling wave functions [9]:

$$\Psi_T = \exp\left(-\frac{1 - [1 - U(\sigma)\hat{a}^2]^{\frac{3}{2}}}{3U(\sigma)}\right), \quad \hat{a}^2U(\sigma) < 1, \quad (18)$$

and

$$\Psi_T = \exp\left(-\frac{1}{3U(\sigma)}\right) \exp\left(\frac{-i[U(\sigma)\hat{a}^2 - 1]^{\frac{3}{2}}}{3U(\sigma)}\right), \quad \hat{a}^2U(\sigma) > 1, \quad (19)$$

where  $\Psi$  for the region  $\hat{a}^2U(\sigma) < 1$  (underbarrier) has the regular behaviour  $\Psi \sim e^{-\frac{\hat{a}^2}{2}}$  for  $\hat{a} \rightarrow 0$  matching with the *nothing* solution (14). We notice that the potential term

$$U(\sigma) = \epsilon F(l) e^{-3b\sigma}$$

plays the role of an effective 4D cosmological term whose properties merit attention. First, the presence of  $\epsilon = \pm 1$  corresponds to positive or negative cosmological term for a given  $F(l)$ . But as was discussed before, only the  $\epsilon = +1$  case is relevant to quantum cosmology with  $k = +1$ , since for  $\epsilon = -1$  there is no maximum (barrier) for the superpotential. Second, the presence of the  $l$ -dependent term  $F(l)$  indicates the contributions of the fifth dimension to the

effective 4D cosmological term. Third, for  $b \gg 1$ , if the parameter  $\sigma$  undergoes time evolution from negative to positive values, then a large cosmological term would become a small one very quickly. This is important, as it is in agreement with quantum tunneling and inflationary ideas where the very early universe might have experienced an extremely inflationary era for  $\sigma < 0$  which was abruptly switched off for  $\sigma > 0$ . This would also be an alternative solution to the well-known cosmological constant problem in that an initially large cosmological term becomes small after the inflationary period.

Considering the function  $F(l)$ , and (4) and (11), we find that a non-zero  $F$  may be achieved by taking linear behaviours for the functions  $\chi$  or  $f$  with respect to  $l$

$$f(l) = \chi(l) = \frac{l}{L}, \quad (20)$$

where a constant  $L$  is introduced to preserve physical dimensions. The choice (20) corresponds to the so-called *canonical metric* [16], and gives

$$F(l) = \frac{2}{l^2}. \quad (21)$$

The cosmological term is then obtained as

$$\Lambda \equiv U(\sigma) = \frac{2}{l^2} e^{-3b\sigma}, \quad (22)$$

with the right dimension of  $(length)^{-2}$ .

One may obtain the probability distribution for the Vilenkin wave function as in [9]:

$$\rho_T \sim \exp \left[ -\frac{2}{3U(\sigma)} \right] \sim \exp \left[ -\frac{l^2 e^{3b\sigma}}{3} \right]. \quad (23)$$

This is maximized for  $\sigma \gg 0$  when  $l \rightarrow 0$ . This condition shows that the 4+1 dimensional universe could have tunneled with a large probability if the fifth dimension was very small, and

that a small 4D universe  $\hat{a}_0 \ll 1$  was born on a 4D hypersurface near to the  $l \simeq 0$  hypersurface<sup>1</sup>.

Let us now consider the Hartle-Hawking wave function for this model. In the presence of matter fields  $\phi$  the Hartle-Hawking wave function of the universe is obtained through the functional integral over Euclidean 4-metrics  $g_{\alpha\beta}$  ( $\alpha, \beta = 0, 1, 2, 3$ ):

$$\Psi(\tilde{h}_{ij}, \tilde{\phi}) = \int_{\mathcal{C}} dg_{\alpha\beta} d\phi \exp[-I(g_{\alpha\beta}, \phi)]. \quad (24)$$

Here the domain  $\mathcal{C}$  is defined by the “no boundary” proposal as all regular compact Euclidean 4-geometries (the boundary of which is  $S^3$  with the induced 3-metric  $\tilde{h}_{i,j}$  ( $i, j = 1, 2, 3$ ) and the regular matter field configurations (the value of which is  $\tilde{\phi}$  on the 3-manifold). However, in our model there are no extra matter fields, other than  $\sigma$  which appears as the dilaton field in the 5-metric. In this way, the corresponding Hartle-Hawking wave function is given as

$$\Psi(\tilde{h}_{\alpha\beta}) = \int_{\mathcal{C}} d\hat{g}_{AB} \exp[-I(\hat{g}_{AB})]. \quad (25)$$

Here  $\hat{g}_{AB}$  ( $A, B = 0, 1, 2, 3, 4$ ) is the 5-metric, and the domain  $\mathcal{C}$  is the class of all regular compact Euclidean 5-geometries whose boundary is  $S^3 \times R$  ( $R$  denotes the fifth non-compact coordinate), with the induced 4-metric  $\tilde{h}_{\alpha\beta}$  ( $\alpha, \beta = 1, 2, 3, 4$ ). On the other hand, the present 5D model is effectively equivalent to a 4D non-minimally coupled dilaton field on  $l = \text{const}$  hypersurfaces. Therefore, using the 5D metric (1), the above integral may be rewritten in its familiar 4D form in the gauge  $\dot{\hat{N}} = 0$  [1] as:

$$\Psi_H(\tilde{a}, \tilde{\sigma})|_{l=\text{const}} = \int d\hat{N} \int \mathcal{D}\hat{a} \mathcal{D}\sigma \exp(-I[\hat{a}(\tau), \sigma(\tau), \hat{N}]). \quad (26)$$

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<sup>1</sup>We recall that although we have taken large values of  $\sigma$  in this approximation to the Wheeler-DeWitt equation (8), the condition  $l \rightarrow 0$  makes it possible to narrow down the superpotential barrier ( $U(\sigma) \gg 1$ ) and tunnel from *nothing* to a small size universe  $\hat{a}_0 = \frac{1}{\sqrt{U(\sigma)}}$ .

Here  $I$  is the Euclidean action for the model,  $\tau$  is the Euclidean time and  $\tilde{a}$  and  $\tilde{\sigma}$  are the final values of  $\hat{a}$  and  $\sigma$  on the 3-geometry  $\tilde{h}_{i,j}$  and  $l = \text{const}$  hypersurface. However, in practice we are usually interested in the semi-classical approximation to the above path integral

$$\Psi_H(\tilde{a}, \tilde{\sigma})|_{l=\text{const}} \approx \exp(-I_{cl}(\tilde{a}, \tilde{\sigma})), \quad (27)$$

where  $I_{cl}(\tilde{a}, \tilde{\sigma})$  is the action for the *instanton* solutions to the Euclidean field equations. The Hartle-Hawking wave function for the model defined by the Lagrangian (3) is well-known and may be obtained by the following transformation [9]:

$$\Psi_H = \Psi_T(U \rightarrow e^{-i\pi}U, \hat{a} \rightarrow e^{i\pi/2}\hat{a}).$$

This yields

$$\Psi_H|_{l=\text{const}} \approx \exp\left(\frac{1 - [1 - U(\sigma)\hat{a}^2]^{\frac{3}{2}}}{3U(\sigma)}\right), \quad \hat{a}^2U(\sigma) < 1, \quad (28)$$

$$\Psi_H|_{l=\text{const}} \approx \exp\left(\frac{1}{3U(\sigma)}\right) \cos\left(\frac{[U(\sigma)\hat{a}^2 - 1]^{\frac{3}{2}}}{3U(\sigma)} - \frac{\pi}{4}\right), \quad \hat{a}^2U(\sigma) > 1. \quad (29)$$

Now, the probability distribution for the Hartle-Hawking wave function as given in [9] is

$$\rho_H \sim \exp\left[\frac{2}{3U(\sigma)}\right] \sim \exp\left[\frac{l^2 e^{3b\sigma}}{3}\right]. \quad (30)$$

Contrary to Vilenkin's case, this probability distribution is maximized for  $l \gg 0$ .

This condition, on the other hand, indicates that a Lorentzian universe was born from a *mother* Euclidean universe with a large probability when the 4D Lorentzian hypersurface was very far from the  $l \simeq 0$  hypersurface.

## 4 Discussion

Usually, in quantum cosmology there is a debate on the choice between Hartle-Hawking and Vilenkin wave functions in concern to the issue of inflation. It is commonly believed that the

Vilenkin wave function leads to inflation. However, for some particular models the Hartle-Hawking wave function claims to predict a period of inflation [2]-[4]. So, as far as inflation is concerned, there is no clear way to decide between the two proposals.

In this paper, we have introduced a new way to compare the two proposals in a higher-dimensional Kaluza-Klein model by using the extra dimension. We have found that if there is a non-compactified extra dimension, the Vilenkin proposal seems more reasonable than the Hartle-Hawking proposal. In the Hartle-Hawking proposal, a large value of  $l$  gives rise to a rather large initial radius of the universe, which seems unnatural. That is, in the Hartle-Hawking proposal there is no good justification for *a big 4D universe which was born on a constant hypersurface  $l \gg 0$* . However, in the Vilenkin proposal the universe starts naturally from a small radius  $\hat{a}_0$  with a large cosmological term  $\Lambda \equiv l^{-2}e^{-3b\sigma}$  ( $0 \ll \sigma < \infty$ ) on the  $l \simeq 0$  hypersurface. Although we have solved the problem in the approximation of large positive values of  $\sigma$ , the result  $l \simeq 0$  hypersurface is expected to remain unchanged for the whole domain  $-\infty < \sigma < \infty$  since  $\sigma$  is independent of  $l$ . Therefore, in the full theory the time evolution  $\sigma(t) < 0 \rightarrow \sigma(t) > 0$  will switch off the large cosmological term. This may be considered as a solution to the cosmological constant problem.

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